Why Did the World Trade Center Collapse?—Simple Analysis

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Abstract: This paper[3] presents a simplified approximate analysis of the overall collapse of the towers of World Trade Center in New York on September 11, 2001. The analysis shows that if prolonged heating caused the majority of columns of a single floor to lose their load carrying capacity, the whole tower was doomed.

Introduction and Failure Scenario

The 110-story towers of the World Trade Center were designed to withstand as a whole the forces caused by a horizontal impact of a large commercial aircraft (Appendix I). So why did a total collapse occur? The cause was the dynamic consequence of the prolonged heating of the steel columns to very high temperature. The heating lowered the yield strength and caused viscoplastic (creep) buckling of the columns of the framed tube along the perimeter of the tower and of the columns in the building core. The likely scenario of failure is approximately as follows.

In stage 1 (Fig. 1), the conflagration caused by the aircraft fuel spilled into the structure causes the steel of the columns to be exposed to sustained temperatures apparently exceeding 800°C. The heating is probably accelerated by a loss of the protective thermal insulation of steel during the initial blast. At such temperatures, structural steel suffers a decrease of yield strength and exhibits significant viscoplastic deformation (i.e., creep—an increase of deformation under sustained load). This leads to creep buckling of columns (e.g., Bažant and Cedolin 1991, Sec. 9), which consequently lose their load carrying capacity (stage 2). Once more than about a half of the columns in the critical floor that is heated most suffer buckling (stage 3), the weight of the upper part of the structure above this floor can no longer be supported, and so the upper part starts falling down onto the lower part below the critical floor, gathering speed until it impacts the lower part. At that moment, the upper part has acquired an enormous kinetic energy and a significant downward velocity. The vertical impact of the mass of the upper part onto the lower part (stage 4) applies enormous vertical dynamic load on the underlying structure, far exceeding its load capacity, even if it is not heated. This causes failure of an underlying multi-floor segment of the tower (stage 4), in which the failure of the connections of the floor-carrying trusses to the columns is either accompanied or quickly followed by buckling of the core columns and overall buckling of the framed tube, with the buckles probably spanning the height of many floors (stage 5, at right), and the upper part possibly getting wedged inside an emptied lower part of the framed tube (stage 5, at left). The buckling is initially plastic but quickly leads to fracture in the plastic hinges. The part of building lying beneath is then impacted again by an even larger mass falling with a greater velocity, and the series of impacts and failures then proceeds all the way down (stage 5).

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Elastic Dynamic Analysis

The details of the failure process after the decisive initial trigger that sets the upper part in motion are of course very complicated and their clarification would require large computer simulations. For example, the upper part of one tower is tilting as it begins to fall (see Appendix II); the distribution of impact forces among the underlying columns of the framed tube and the core, and between the columns and the floor-supporting trusses, is highly nonuniform; etc. However, a computer is not necessary to conclude that the collapse of the majority of columns of one floor must have caused the whole tower to collapse. This may be demonstrated by the following elementary calculations, in which simplifying assumptions most optimistic in regard to survival are made.

For a short time after the vertical impact of the upper part, but after the elastic wave generated by the vertical impact has propagated to the ground, the lower part of the structure can be approximately considered to act as an elastic spring (Fig. 2a). What is its stiffness C? It can vary greatly with the distribution of the impact forces among the framed tube columns, between these columns and those in the core, and between the columns and the trusses supporting concrete floor slabs.

For our purpose, we may assume that all the impact forces go into the columns and are distributed among them equally. Unlikely though such a distribution may be, it is nevertheless the most optimistic hypothesis to make because the resistance of the building to the impact is, for such a distribution, the highest. If the building is found to fail under a uniform distribution of the impact forces, it would fail under any other distribution. According to this hypothesis, one may estimate that $C \approx 71$ GN/m (due to unavailability of precise data, an approximate design of column cross sections had to be carried out for this purpose).

The downward displacement from the initial equilibrium position to the point of maximum deflection of the lower part (considered to behave elastically) is $h + (P/C)$ where P = maximum force applied by the upper part on the lower part and h = height of critical floor columns (= height of the initial fall of the upper part) ≈ 3.7 m. The energy dissipation, particularly that due to the inelastic deformation of columns during the initial drop of the upper part, may be neglected, i.e., the upper part may be assumed to move through distance h almost in a free fall (indeed, the energy dissipated in the columns during the fall is at most equal to $2\pi\times$ the yield moment of columns, $\times$ the number of columns, which is found to be only about 12% of the gravitational potential energy release if the columns were cold, and much less than that at 800°C). So the loss of the gravitational potential energy of the upper part may be approximately equated to the strain energy of the lower part at maximum elastic deflection. This gives the equation $mg[h + (P/C)] = P^2/2C$ in which m = mass of the upper part (of North Tower) $\approx 58{\cdot}10^6$ kg, and g = gravity acceleration. The solution $P = P_{dyn}$ yields the following elastically calculated overload ratio due to impact of the upper part:

$$P_{dyn}/P_0 \;=\; 1 + \sqrt{1 + (2Ch/mg)} \;\approx\; 31 \tag{1}$$

where $P_0 = mg$ = design load capacity. In spite of the approximate nature of this analysis, it is obvious that the elastically calculated forces in columns caused by the vertical impact of the upper part must have exceeded the load capacity of the lower part by at least an order of magnitude.

Another estimate, which gives the initial overload ratio that exists only for a small fraction of a second at the moment of impact, is

$$P_{dyn}/P_0 \;=\; (A/P_0)\sqrt{2\rho g E_{ef} h} \;\approx\; 64.5 \tag{2}$$

where A = cross section area of building, E_{ef} = cross section stiffness of all columns divided by A, ρ = specific mass of building per unit volume. This estimate is calculated from the elastic wave equation which yields the intensity of the step front of the downward pressure wave caused by the impact if the velocity of the upper part at the moment of impact on the critical floor is considered as the boundary condition (e.g., Bažant and Cedolin, Sec. 13.1). After the wave propagates to the ground, the former estimate is appropriate.

Analysis of Inelastic Energy Dissipation

The inelastic deformation of the steel of the towers involves plasticity and fracture. Since we are not attempting to model the details of the real failure mechanism but seek only to prove that the towers must have collapsed and do so in the way seen (Engineering 2001, American 2001), we will here neglect fracture, even though the development of fractures is clearly discerned in the photographs of the collapse. Assuming the steel to behave plastically, with unlimited ductility, we are making the most optimistic assumption with regard to the survival capacity of the towers (in reality, the plastic hinges, especially the hinges at column connections, must have fractured, and done so at relatively small rotation, causing the load capacity to drop drastically).

The basic question to answer is: Can the fall of the upper part be arrested by energy dissipation during plastic buckling which follows the initial elastic deformation? Many plastic failure mechanisms could be considered, for example: (a) the columns of the underlying floor buckle locally (Fig. 1, stage 2); (b) the floor-supporting trusses are sheared off at the connections to the framed tube and the core columns and fall down within the tube, depriving the core columns and the framed tube of lateral support, and thus promoting buckling of the core columns and the framed tube under vertical compression (Fig. 1, stage 4, Fig. 2c); or (c) the upper part is partly wedged within the emptied framed tube of the lower part, pushing the walls of the framed tube apart (Fig. 1, stage 5). Although each of these mechanism can be shown to lead to total collapse, a combination of the last two seems more realistic (the reason: multi-story pieces of the framed tube, with nearly straight boundaries apparently corresponding to plastic hinge lines causing buckles on the framed tube wall, were photographed falling down; see, e.g., Engineering 2001, American 2001).

Regardless of the precise failure mode, experience with buckling indicates that the while many elastic buckles simultaneously coexist in an axially compressed tube, the plastic deformation localizes (because of plastic bifurcation) into a single buckle at a time (Fig. 1, stage 4; Fig. 2c), and so the buckles must fold one after another. Thus, at least one plastic hinge, and no more than four plastic hinges, per column line are needed to operate simultaneously in order to allow the upper part to continue moving down (Fig. 2b, Bažant and Cedolin 1991) (this is also true if the columns of only one floor are buckling at a time). At the end, the sum of the rotation angles θ_i $(i = 1, 2, ..)$ of the hinges on one column line, $\sum \theta_i$, cannot exceed 2π (Fig. 2b). This upper-bound value, which is independent of the number of floors spanned by the buckle, is used in the present calculations since, in regard to survival, it represents the most optimistic hypothesis, maximizing the plastic energy dissipation.

Calculating the dissipation per column line of the framed tube as the plastic bending moment M_p of one column (Jirásek and Bažant 2002), times the combined rotation angle $\sum \theta_i = 2\pi$ (Fig. 2b), and multiplying this by the number of columns, one concludes that the plastically dissipated energy W_p is, optimistically, of the order of 0.5 GN m (for lack of information, certain details such as the wall thickness of steel columns, were estimated by carrying out approximate design calculations for this building).

To attain the combined rotation angle $\sum \theta_i = 2\pi$ of the plastic hinges on each column line, the upper part of the building must move down by the additional distance of one buckle, which is *at least* one floor below the floor where the collapse started. So the additional release of gravitational potential energy $W_g \geq mg \cdot 2h \approx 2 \times 2.1$ GN m $= 4.2$ GN m. To arrest the fall, the kinetic energy of the upper part, which is equal to the potential energy release for a fall through the height of at least two floors, would have to be absorbed by the plastic hinge rotations of one buckle, i.e., W_g/W_p would have to be less than 1. Rather,

$$W_g/W_p \approx 8.4 \tag{3}$$

if the energy dissipated by the columns of the critical heated floor is neglected. If the first buckle spans over n floors (3 to 10 seems likely), this ratio is about n times larger. So, even under by far the most optimistic assumptions, the plastic deformation can dissipate only a small part of the kinetic energy acquired by the upper part of building.

When the next buckle with its group of plastic hinges forms, the upper part has already traveled many floors down and has acquired a much higher kinetic energy; the percentage of the kinetic energy dissipated plastically is then of the order of 1%. The percentage continues to decrease further as the upper part moves down. If fracturing in the plastic hinges were considered, a still smaller (in fact much smaller) energy dissipation would be obtained. So the collapse of the tower must be an almost free fall. This conclusion is supported by the observation that the duration of the collapse of the tower, observed to be 9 s, was about the same as the duration of a free fall in a vacuum from the tower top (416 m above ground) to the top of the final heap of debris (about 25 m above ground), which is $t = \sqrt{2\ (416\text{m} - 25\text{m})/g} = 8.93$ s. It further follows that the brunt of vertical impact must have gone directly into the columns of the framed tube and the core and that any delay Δt of the front of collapse of the framed tube behind the front of collapsing ('pancaking') floors must have been negligible, or else the duration of the total collapse of the tower, 9 s + Δt, would have been significantly longer than 9 s. However, even for a short delay Δt, the floors should have acted like a piston running down through an empty tube, which helps to explain the smoke and debris that was seen being expelled laterally from the collapsing tower.

Problems of Disaster Mitigation and Design

Designing tall buildings to withstand this sort of attack seems next to impossible. It would require a much thicker insulation of steel, with blast-resistant protective cover. Replacing the rectangular framed tube by a hardened circular monolithic tube with tiny windows might help to deflect much of the debris and fuel from an impacting aircraft sideways, but regardless of cost, who would want to work in such a building?

The problems appear to be equally severe for concrete columns because concrete heated to such temperatures undergoes explosive thermal spalling, thermal fracture and disintegration due to dehydration of hardened cement paste (e.g., Bažant and Kaplan 1996). These questions arise not only for buildings supported on many columns but also for the recent designs of tall buildings with a massive monolithic concrete core functioning as a tubular mast. These recent designs use high-strength concrete which, however, is even more susceptible to explosive thermal spalling and thermal fracture than normal concrete. The use of refractory concretes as the structural material invites many open questions (Bažant and Kaplan 1996). Special alloys or various refractory ceramic composites may of course function at such temperatures, but the cost would increase astronomically.

It will nevertheless be appropriate to initiate research on materials and designs that would postpone the collapse of the building so as to extend the time available for evacuation, provide a hardened and better insulated stairwell, or even prevent collapse in the case of a less severe attack such as an off-center impact or the impact of an aircraft containing little fuel.

Lessons should be drawn for improving the safety of building design in the case of lesser disasters. For instance, in view of the progressive dynamic collapse of a stack of all the floors of the Ronan Point apartments in the U.K., caused by a gas explosion in one upper floor (Levy and Salvadori 1992), the following design principle, determining the appropriate degree of redundancy, should be adopted: If only a certain judiciously specified minority of the columns or column-floor connections at one floor are removed, the mass that might fall down from the superior structure must be so small that its impact on the underlying structure would not cause dynamic overload.

Closing Comments

Once accurate computer calculations are carried out, various details of the failure mechanism will doubtless be found to differ from the present simplifying hypotheses. Errors by a factor of 2 would not be terribly surprising, but that would hardly matter since the present analysis reveals order-of-magnitude differences between the dynamic loads and the structural resistance.

There have been many interesting, but intuitive, competing explanations of the collapse. To decide their viability, however, it is important to do at least some crude calculations. For example, it has been suggested that the connections of the floor-supporting trusses to the framed tube columns were not strong enough. Maybe they were not, but even if they were it would have made no difference, as shown by the present simple analysis.

The main purpose of the present analysis is to prove that the whole tower must have collapsed if the fire destroyed the load capacity of the majority of columns of a single floor. This purpose justifies the optimistic simplifying assumptions regarding survival made at the outset, which include unlimited plastic ductility (i.e., absence of fracture), uniform distribution of impact forces among the columns, disregard of various complicating details (e.g., the possibility that the failures of floor-column connections and of core columns preceded the column and tube failure, or that the upper tube got wedged inside the lower tube), etc. If the tower is found to fail under these very optimistic assumptions, it will certainly be found to fail when all the detailed mechanisms are analyzed, especially since there are order-of-magnitude differences between the dynamic loads and the structural resistance.

An important puzzle at the moment is why the adjacent 46-story building, into which no significant amount of aircraft fuel could have been injected, collapsed as well. Despite the lack of data at present, the likely explanation seems to be that high temperatures (though possibly well below 800°C) persisted on at least one floor of that building for a much longer time than specified by the current fire code provisions.

Appendix I. Elastic Dynamic Response to Aircraft Impact

A simple estimate based on the preservation of the combined momentum of the impacting Boeing 767-200 ($\sim$ 179,000 kg $\times$ 550 km/h) and the momentum of the equivalent mass M_{eq} of the interacting upper part of the tower ($\sim 141 \cdot 10^6 \times v_0$) indicates that the initial average velocity v_0 imparted to the upper part of the tower was only about 0.7 km/h = 0.19 m/s. Mass M_{eq}, which is imagined as a concentrated mass mounted at the height of the impacted floor on a massless free-standing cantilever with the same bending stiffness as the tower (Fig. 2d), has been calculated from the condition that its free vibration period be equal to the first vibration period of the tower, which has been roughly estimated as $T_1 = 14$ s ($M_{eq} \approx 44\%$ of the mass of the whole tower). The dynamic response after impact may be assumed to be dominated by the first free vibration mode, of period T_1. Therefore, the maximum horizontal deflection $w_0 = v_0 T_1 / 2\pi \approx 0.4$ m, which is well within the design range of wind-induced elastic deflections. So it is not surprising that the aircraft impact per se damaged the tower only locally.

The World Trade Center was designed for an impact of Boeing 707-320 rather than Boeing 767-320. But note that the maximum takeoff weight of that older, less efficient, aircraft is only 15% less than that of Boeing 767-200. Besides, the maximum fuel tank capacity of that aircraft is only 4% less. These differences are well within the safety margins of design. So the observed response of the towers proves the correctness of the original dynamic design. What was not considered in design was the temperature that can develop in the ensuing fire. Here the lulling experience from 1945 might have been deceptive; that year, a two-engine bomber (B-25), flying in low clouds to Newark at about 400 km/h, hit the Empire State Building (381 m tall, built in 1932) at the 79th floor (278 m above ground)—the steel columns (much heavier than in modern buildings) suffered no significant damage, and the fire remained confined essentially to two floors only (Levy and Salvadori 1992).

Appendix II. Why Didn't the Upper Part Pivot About Its Base?

Since the top part of the South Tower tilted (Fig. 3a), many people wonder: Why didn't the upper part of the tower fall to the side like a tree, pivoting about the center of the critical floor? (Fig. 3b) To demonstrate why, and thus to justify our previous neglect of tilting, is an elementary exercise in dynamics.

Assume the center of the floor at the base of the upper part (Fig. 3b) to move for a while neither laterally nor vertically, i.e., act as a fixed pivot. Equating the kinetic energy of the upper part rotating as a rigid body about the pivot at its base (Fig. 3c) to the loss of the gravitational potential energy of that part (which is here simpler than using the Lagrange equations of motion), we have $mg(1 - \cos\theta)H_1/2 = (m/2H_1)\int_0^{H_1}(\dot{\theta}x)^2 \, \mathrm{d}x$ where x is the vertical coordinate (Fig. 3c).

This provides

$$\dot{\theta} = \sqrt{\frac{3g}{H_1}(1 - \cos\theta)}, \qquad \ddot{\theta} = \frac{3g}{2H_1}\ \sin\theta \tag{4}$$

where θ = rotation angle of the upper part, H_1 = its height, and the superposed dots denote time derivatives (Fig. 3c).

Considering the dynamic equilibrium of the upper part as a free body, acted upon by distributed inertia forces and a reaction with horizontal component F at base (Fig. 3d), one obtains $F = \int_0^{H_1}(m/H_1)\ \ddot{\theta}\ \cos\theta\ x\mathrm{d}x = \frac{1}{2}\ H_1 m\ \ddot{\theta}\ \cos\theta =\ \frac{3}{8}mg\ \sin 2\theta$. Evidently, the maximum horizontal reaction during pivoting occurs for $\theta = 45°$, and so

$$F_{max}\ =\ \tfrac{3}{8}\ mg\ =\ \tfrac{3}{8}\ P_0\ \approx\ 320\ \text{MN} \tag{5}$$

where, for the upper part of South Tower, $m \approx 87 \cdot 10^6$ kg.

Could the combined plastic shear resistance F_p of the columns of one floor (Fig. 3f) sustain this horizontal reaction? For plastic shear, there would be yield hinges on top and bottom of each resisting column; Fig. 3e (again, aiming only at an optimistic upper bound on resistance, we neglect fracture). The moment equilibrium condition for the column as a free body shows that each column can at most sustain the shear force $F_1 = 2M_p/h_1$ where $h_1 \approx 2.5$ m = effective height of column, and $M_p \approx 0.3$ MN m = estimated yield bending moment of one column, if cold. Assuming that the resisting columns are only those at the sides of the framed tube normal to the axis of rotation, which number about 130, we get $F_p \approx 130F_1 \approx 31$ MN. So, the maximum horizontal reaction to pivoting would cause the overload ratio

$$F_{max}/F_p\ \approx\ 10.3 \tag{6}$$

if the resisting columns were cold. Since they are hot, the horizontal reaction to pivoting would exceed the shear capacity of the heated floor still much more (and far more if fracture were considered).

Since F is proportional to $\sin 2\theta$, its value becomes equal to the plastic limit when $\sin 2\theta = 1/10.3$. From this we further conclude that the reaction at the base of the upper part of South Tower must have begun shearing the columns plastically already at the inclination

$$\theta\ \approx\ 2.8° \tag{7}$$

The pivoting of the upper part must have started by an asymmetric failure of the columns on one side of building, but already at this very small angle the dynamic horizontal reaction at the base of the upper part must have reduced the vertical load capacity of the remaining columns of the critical floor (even if those were not heated). That must have started the downward motion of the top part of the South Tower, and afterwards its motion must have become predominantly vertical. Hence, a vertical impact of the upper part onto the lower part must have been the dominant mechanism.

Finally note that the horizontal reaction F_{max} is proportional to the weight of the pivoting part. Therefore, if a pivoting motion about the center of some lower floor were considered, F_{max} would be still larger.

Appendix III. Plastic Load-Shortening Diagram of Columns

Normal design deals only with initial bifurcation and small deflections, in which the diagram of load versus axial shortening of an elasto-plastic column exhibits hardening rather than softening. However, the columns of the towers suffered very large plastic deflections, for which this diagram exhibits pronounced softening. Fig. 5 shows this diagram as estimated for these towers. The diagram begins with axial shortening due to plastic yielding at load $P_1^0 = A_1 f_y$ where A_1 = cross-section area of one column and f_y = yield limit of steel. At the axial shortening of about 3%, there is a plastic bifurcation (if imperfections are ignored). After that, undeflected states are unstable and three plastic hinges (Fig. 5) must form (if we assume, optimistically, the ends to be fixed). From the condition of moment equilibrium of the half-column as a free body (Fig. 5), the axial load then is $P_1 = 4M_p/L \sin\theta$, while, from the buckling geometry, the axial shortening is $u = L(1 - \cos\theta)$, where L = distance between the end hinges. Eliminating plastic hinge rotation θ, we find that the plastic load-shortening diagram (including the pre- and post-bifurcation states) is given by

$$P_1(u) \ = \ \min\left(\ \frac{4M_p}{L\sqrt{1-[1-(u/L)]^2}} \ , \ \ P_1^0 \right) \tag{8}$$

which defines the curve plotted in Fig. 5. This curve is an optimistic upper bound since, in reality, the plastic hinges develop fracture (e.g., Bažant and Planas 1998), and probably do so already at rather small rotations. The area under this curve represents the dissipated energy.

If it is assumed that one or several floor slabs above the critical heated floor collapsed first, then the L to be substituted in (8) is much longer than the height of columns of one floor. Consequently, $P_1(u)$ becomes much smaller (and the Euler elastic critical load for buckling may even become less than the plastic load capacity, which is far from true when L is the column height of a single floor).

It has been suggested that the inelastic deformation of columns might have 'cushioned' the initial descent of the upper part, making it almost static. However, this is impossible because, for gravity loading, a softening of the load-deflection diagram (Fig. 5) always causes instability and precludes static deformation (Bažant and Cedolin 1991, Chpt. 10 and 13). The downward acceleration of the upper part is $\ddot{u} = N[P_1^0 - P_1(u)]/m$ where N = number of columns and, necessarily, $P_1^0 = mg/N$. This represents a differential equation for u as a function of time t, and its integration shows that the time that the upper part takes to fall through the height of one story is, for cold columns, only about 6% longer than the duration of a free fall from that height, which is 0.87 s. For hot columns, the difference is of course much less than 6%. So there is hardly any 'cushioning'.

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Captions:

Fig. 1 Stages of collapse of the building (floor height exaggerated).

Fig. 2 (a) Model for impact of upper part on lower part of building. (b) Plastic buckling mechanism on one column line. (c) Combination of plastic hinges creating a buckle in the tube wall. (d) Equivalent mass M_{eq} on a massless column vibrating at the same frequency.

Fig. 3 Pivoting of upper part of tower about its base; (a,b) with and without horizontal shear at base; (c) model for simplified analysis; (d) free-body diagram with inertia forces; (d,e) plastic horizontal shearing of columns in critical floor at base.

Fig. 4 Scenario of tilting of upper part of building (South Tower).

Fig. 5 (a) Plastic buckling of columns; (b) plastic hinge mechanism; (b) free-body diagram; (d) dimensionless diagram of load P_1 versus axial shortening u of columns of the towers if the effects of fracture and heating are ignored; and (e) the beginning of this diagrams in an expanded horizontal scale (imperfections neglected).