Mechanics of Progressive Collapse: Learning from World Trade Center and Building Demolitions

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Abstract: Progressive collapse is a failure mode of great concern for tall buildings, and is also typical of building demolitions. The most infamous paradigm is the collapse of the World Trade Center towers. After reviewing the mechanics of their collapse, the motion during the crushing of one floor (or group of floors) and its energetics are analyzed, and a dynamic one-dimensional continuum model of progressive collapse is developed. Rather than using classical homogenization, it is found more effective to characterize the continuum by an energetically equivalent snap-through. The collapse, in which two phases—crush-down followed by crush-up—must be distinguished, is described in each phase by a nonlinear second-order differential equation for the propagation of the crushing front of a compacted block of accreting mass. Expressions for consistent energy potentials are formulated and an exact analytical solution of a special case is given. It is shown that progressive collapse will be triggered if the total (internal) energy loss during the crushing of one story (equal to the energy dissipated by the complete crushing and compaction of one story, minus the loss of gravity potential during the crushing of that story) exceeds the kinetic energy impacted to that story. Regardless of the load capacity of the columns, there is no way to deny the inevitability of progressive collapse driven by gravity *alone* if this criterion is satisfied (for the World Trade Center it is satisfied with an order-of-magnitude margin). The parameters are the compaction ratio of a crushed story, the fracture of mass ejected outside the tower perimeter, and the energy dissipation per unit height. The last is the most important, yet the hardest to predict theoretically. It is argued that, using inverse analysis, one could identify these parameters from a precise record of the motion of floors of a collapsing building. Due to a shroud of dust and smoke, the videos of the World Trade Center are only of limited use. It is proposed to obtain such records by monitoring (with millisecond accuracy) the precise time history of displacements in different modes of building demolitions. The monitoring could be accomplished by real-time telemetry from sacrificial accelerometers, or by high-speed optical camera. The resulting information on energy absorption capability would be valuable for the rating of various structural systems and for inferring their collapse mode under extreme fire, internal explosion, external blast, impact or other kinds of terrorist attack, as well as earthquake and foundation movements.

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Introduction

The destruction of the World Trade Center (WTC) on September 11, 2001 was not only the largest mass murder in U.S. history but also a big surprise for the structural engineering profession, perhaps the biggest since the collapse of the Tacoma Bridge in 1940. No experienced structural engineer watching the attack expected the WTC towers to collapse. No skyscraper has ever before collapsed due to fire. The fact that the WTC towers did, beckons deep examination.

In this paper [based on Bažant and Verdure's (2006) identical report presented at the U.S. National Congress of Theoretical and Applied Mechanics, Boulder, Colo., June 26, 2006; and posted on June 23, 2006, at www.civil.northwestern.edu/people/bazant.html], attention will be focused on the progressive collapse, triggered in the WTC by fire and previously experienced in many tall buildings as a result of earthquake or explosions (including terrorist attack). A simplified one-dimensional analytical solution of the collapse front propagation will be presented. It will be shown how this solution can be used to determine the energy absorption capability of individual stories if the motion history is precisely recorded. Because of the shroud of dust and smoke, these histories can be identified from the videos of the collapsing WTC towers only for the first few seconds of collapse, and so little can be learned in this regard from that collapse. However, monitoring of tall building demolitions, which represent one kind of progressive collapse, could provide such histories. Development of a simple theory amenable to inverse analysis of these histories is the key. It would permit extracting valuable information on the energy absorption capability of various types of structural systems in various collapse modes, and is, therefore, the main objective of this paper.

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Many disasters other than the WTC attest to the danger of progressive collapse, e.g., the collapse of Ronan Point apartments in the United Kingdom in 1968 (Levy and Salvadori 1992), where a kitchen gas explosion on the 18th floor sent a 25-story stack of rooms to the ground; the bombing of the Murrah Federal Building in Oklahoma City, Okla., in 1995, where the air blast pressure sufficed to take out only a few lower floors, whereas the upper floors failed by progressive collapse; the 2000 Commonwealth Ave. tower in Boston in 1971, triggered by punching of insufficiently hardened slab; the New World Hotel in Singapore; many buildings in Armenia, Turkey, Mexico City, and other earthquakes, etc. A number of ancient towers failed in this way, too, e.g., the Civic Center of Pavia in 1989 (Binda et al. 1992); the cathedral in Goch, Germany; the Campanile in Venice in 1902, etc. (Heinle and Leonhardt 1989), where the trigger was centuries-long stress redistribution due to drying shrinkage and creep (Ferretti and Bažant 2006a,b).

Review of Causes of WTC Collapse

Although the structural damage inflicted by aircraft was severe, it was only local. Without stripping of a significant portion of the steel insulation during impact, the subsequent fire would likely not have led to overall collapse (Bažant and Zhou 2002a; NIST 2005). As generally accepted by the community of specialists in structural mechanics and structural engineering (though not by a few outsiders claiming a conspiracy with planted explosives), the failure scenario was as follows:

1. About 60% of the 60 columns of the impacted face of framed tube (and about 13% of the total of 287 columns) were severed, and many more were significantly deflected. This caused stress redistribution, which significantly increased the load of some columns, attaining or nearing the load capacity for some of them.
2. Because a significant amount of steel insulation was stripped, many structural steel members heated up to 600°C, as confirmed by annealing studies of steel debris (NIST 2005) [the structural steel used loses about 20% of its yield strength already at 300°C, and about 85% at 600°C (NIST 2005); and exhibits significant viscoplasticity, or creep, above 450°C (e.g., Cottrell 1964, p. 299), especially in the columns overstressed due to load redistribution; the press reports right after September 11, 2001 indicating temperature in excess of 800°C, turned out to be groundless, but Bažant and Zhou's analysis did not depend on that].
3. Differential thermal expansion, combined with heat-induced viscoplastic deformation, caused the floor trusses to sag. The catenary action of the sagging trusses pulled many perimeter columns inward (by about 1 m, NIST 2005). The bowing of these columns served as a huge imperfection inducing multistory out-of-plane buckling of framed tube wall. The lateral deflections of some columns due to aircraft impact, the differential thermal expansion, and overstress due to load redistribution also diminished buckling strength.
4. The combination of seven effects—(1) Overstress of some columns due to initial load redistribution; (2) overheating due to loss of steel insulation; (3) drastic lowering of yield limit and creep threshold by heat; (4) lateral deflections of many columns due to thermal strains and sagging floor trusses; (5) weakened lateral support due to reduced in-plane stiffness of sagging floors; (6) multistory bowing of some columns (for which the critical load is an order of magnitude less than it is for one-story buckling); and (7) local plastic buckling of heated column webs—finally led to buckling of columns [Fig. 1(b)]. As a result, the upper part of the tower fell, with little resistance, through at least one floor height, impacting the lower part of the tower. This triggered progressive collapse because the kinetic energy of the falling upper part exceeded (by an order of magnitude) the energy that could be absorbed by limited plastic deformations and fracturing in the lower part of the tower.

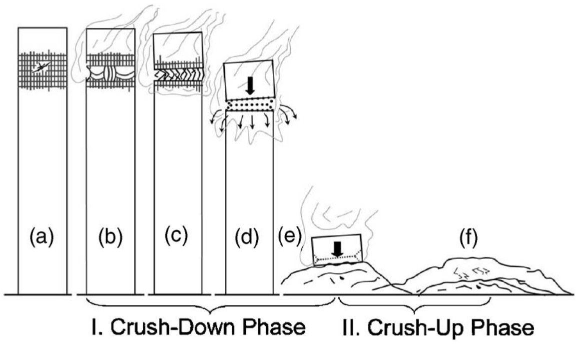

Fig. 1. Scenario of progressive collapse of the World Trade Center towers

In broad terms, this scenario was proposed by Bažant (2001), and Bažant and Zhou (2002a,b) on the basis of simplified analysis relying solely on energy considerations. Up to the moment of collapse trigger, the foregoing scenario was identified by meticulous, exhaustive, and very realistic computer simulations of unprecedented detail, conducted by S. Shyam Sunder's team at NIST. The subsequent progressive collapse was not simulated at NIST because its inevitability, once triggered by impact after column buckling, had already been proven by Bažant and Zhou's (2002a) comparison of kinetic energy to energy absorption capability. The elastically calculated stresses caused by impact of the upper part of tower onto the lower part were found to be 31 times greater than the design stresses (note a misprint in Eq. 2 of Bažant and Zhou 2002a: *A* should be the combined cross section area of all columns, which means that Eq. 1, rather than 2, is decisive).

Before disappearing from view, the upper part of the South tower was seen to tilt significantly (and of the North tower mildly). Some wondered why the tilting [Fig. 1(d)] did not continue, so that the upper part would pivot about its base like a falling tree [see Fig. 4 of Bažant and Zhou (2002b]. However, such toppling to the side was impossible because the horizontal reaction to the rate of angular momentum of the upper part would have exceeded the elastoplastic shear resistance of the story at least 10.3× (Bažant and Zhou 2002b).

The kinetic energy of the top part of the tower impacting the floor below was found to be about 8.4× larger than the plastic energy absorption capability of the underlying story, and considerably higher than that if fracturing were taken into account (Bažant and Zhou 2002a). This fact, along with the fact that during the progressive collapse of underlying stories [Figs. 1(d) and 2] the loss of gravitational potential per story is much greater than the energy dissipated per story, was sufficient for Bažant and Zhou (2002a) to conclude, purely on energy grounds, that the tower was doomed once the top part of the tower dropped through the height of one story (or even 0.5 m). It was also observed that this conclusion made any calculations of the dynamics of progressive collapse after the first single-story drop of upper part superfluous. The relative smallness of energy absorption capability

$$(f\dot{f})^{\cdot} = Qf - P \tag{31}$$

Here $Q=1/(1-\lambda)$, $P(t)=F_c/\mu(1-\lambda)gH$, $F_c=F_c[z(t)]$, $f=f(t)=z(t)/H$; and the superior dots now denote derivatives with respect to dimensionless time $\tau=t\sqrt{g/H}$. Let $\varphi=f^2/2$. Then $\dot{\varphi}=f\dot{f}$ and

$$\ddot{\varphi} = Q\sqrt{2\varphi} - P \tag{32}$$

$$(\dot{\varphi}^2)^{\cdot} = 2\dot{\varphi}\ddot{\varphi} = 2(Q\sqrt{2\varphi} - P)\dot{\varphi} \tag{33}$$

$$\int \mathrm{d}(\dot{\varphi}^2) = \int 2(Q\sqrt{2\varphi} - P)\mathrm{d}\varphi \tag{34}$$

$$\dot{\varphi} = \left(\frac{4}{3}Q\sqrt{2}\varphi^{3/2} - 2P\varphi + C\right)^{1/2} \tag{35}$$

$$\tau - \tau_0 = \int_{\varphi(\tau_0)}^{\varphi(\tau)} \left(\frac{4}{3}Q\sqrt{2}\varphi^{3/2} - 2P\varphi + C\right)^{-1/2} \mathrm{d}\varphi \tag{36}$$

The second equation was obtained by multiplying the first by $2\dot{\varphi}$, and Eq. (35) was integrated by separation of variables; C and τ_0 are integration constants defined by the initial conditions. The last equation describes the collapse history parametrically; for any chosen φ, it yields the time as $t=z\sqrt{H/g}$ or $y\sqrt{H/g}$ where z or $y=H\sqrt{2\varphi}$.

Eq. (12) for the crush-up phase with constant μ and λ takes the form

$$f\ddot{f} + Qf = P \tag{37}$$

Multiplying this equation by $\dot{f}/f$ and noting that $\dot{f}\ddot{f}=(1/2)(\dot{f}^2)^{\cdot}$ and $\dot{f}/f=(\ln f)^{\cdot}$, one may get the solution as follows:

$$(\dot{f}^2)^{\cdot} = 2(P\dot{f}/f - Q\dot{f}) \tag{38}$$

$$\dot{f}^2 = 2(P\ln f - Qf) + C \tag{39}$$

$$\mathrm{d}f = [2(P\ln f - Qf) + C]^{1/2}\mathrm{d}\tau \tag{40}$$

$$\tau - \tau_0 = \int_{f(\tau_0)}^{f(\tau)} [2(P\ln f - Qf) + C]^{-1/2}\mathrm{d}\tau \tag{41}$$

Effect of Elastic Waves

The elastic part of the response did not have to be included in Eqs. (12) and (17) because it cannot appreciably interfere with the buckling and crushing process. The reason is that, at the limit of elasticity of steel, the shortening of story height is only about $h/500$, and the elastic wave in steel is about 600× faster than the crushing front at $z=z_0$. An elastic stress wave with approximately step wave front and stress not exceeding the yield limit of steel emanates from the crushing front when each floor is hit, propagates down the tower, reflects from the ground, etc. But the damage to the tower is almost nil because the stress in the wave must remain in the elastic range and the perfectly plastic part of steel deformation cannot propagate as a wave (Goldsmith 2001; Zukas et al. 1982; Cristescu 1972; Kolsky 1963).

Analogous Problem—Crushing of Foam

A rigid foam is homogenized by a nonlocal strain-softening continuum. Pore collapse represents a localization instability which cannot propagate by itself. But it can if driven by inertia of an impacting object or by blast pressure. One-dimensional impact crushing can be easily solved from Eq. (12) if the top part of the tower is replaced by a rigid impacting object of a mass equivalent to $m(z_0)$, the initial velocity of which is assigned as the initial condition at $t=0$. Compared to inertia forces, gravity may normally be neglected (i.e., $g=0$).

Implications and Conclusions

1. If the total (internal) energy loss during the crushing of one story (representing the energy dissipated by the complete crushing and compaction of one story, minus the loss of gravity potential during the crushing of that story) exceeds the kinetic energy impacted to that story, collapse will continue to the next story. This is the criterion of progressive collapse trigger [Eq. (5)]. If it is satisfied, there is no way to deny the inevitability of progressive collapse driven by gravity *alone* (regardless of by how much the combined strength of columns of one floor may exceed the weight of the part of the tower above that floor). What matters is energy, not the strength, nor stiffness.
2. One-dimensional continuum idealization of progressive collapse is amenable to a simple analytical solution which brings to light the salient properties of the collapse process. The key idea is not to use classical homogenization, leading to a softening stress-strain relation necessitating nonlocal finite element analysis, but to formulate a continuum energetically equivalent to the snapthrough of columns.
3. Distinction must be made between crush-down and crush-up phases, for which the crushing front of a moving block with accreting mass propagates into the stationary stories below, or into the moving stories above, respectively. This leads to a second-order nonlinear differential equation for propagation of the crushing front, which is different for the crush-down phase and the subsequent crush-up phase.
4. The mode and duration of collapse of WTC towers are consistent with the present model, but not much could be learned because, after the first few seconds, the motion became obstructed from view by a shroud of dust and smoke.
5. The present idealized model allows simple inverse analysis which can yield the crushing energy per story and other properties of the structure from a precisely recorded history of motion during collapse. From the crushing energy, one can infer the collapse mode, e.g., single-story or multistory buckling of columns.
6. It is proposed to monitor the precise time history of displacements in building demolitions—for example, by radio telemetry from sacrificial accelerometers, or high-speed optical camera—and to engineer different modes of collapse to be monitored. This should provide invaluable information on the energy absorption capability of various structural systems, needed for assessing the effects of explosions, impacts, earthquake, and terrorist acts.

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